13-1 Preference Tables and the Plurality Method

Preference Tables

Preference ballot – a ballot where, instead of voting for a single candidate, you are asked to rank each candidate in order of preference.
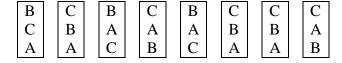
Ex:

B	B	B	B
C	A	A	A
A	C	C	C

Plurality method – in an election with three or more candidates, the candidate with the most first-place votes is the winner.

Ex. A club was voting for a new president from the following candidates: Andy (A), Bella (B), and Colin (C). Make a preference table for the following preference ballots.

B	B	B	B	B	B	B	B	C	B	B	C	B	C
C	A	A	A	C	A	C	A	B	C	C	A	A	B
A	C	C	C	A	C	A	C	A	A	A	B	C	A

B	C	B	C	B	C	C	C
C	B	A	A	A	B	B	A
A	A	C	B	C	A	A	B

(a) How many people voted?
(b) How many people voted for the candidates in the order of preference BCA?
(c) How many people voted for Colin as their first choice?
(d) Using the plurality method, determine the winner of the election.

Answer: Preference Table:

Number of votes	8	6	3	5
First choice	B	B	C	C
Second choice	A	C	A	B
Third choice	C	A	B	A

(a) 22
(b) 6
(c) 3 + 5 = 8
(d) Bella is the winner with 14 votes (8 + 6)

Head-to-Head Comparison Criterion

❖ If a particular candidate wins all head-to-head comparisons with all other candidates, then the candidate should win the election.

Ex. Did the election in the previous example violate the head-to-head comparison criterion?

B preferred over A: 1st column (8), 2nd column (6), 4th column (5)

$8 + 6 + 5 = 19$ voters prefer B over A

A preferred over B: 3rd column (3)

3 voters prefer A over B

B wins over A

B preferred over C: 1st column (8), 2nd column (6)

$8 + 6 = 14$ voters prefer B over C

C preferred over B: 3rd column (3), 4th column (5)

$3 + 5 = 8$ voters prefer C over B

B wins over C

A preferred over C: 1st column (8)

8 voters prefer A over C

C preferred over A: 2nd column (6), 3rd column (3), 4th column (5)

$6 + 3 + 5 = 14$ voters prefer C over A

C wins over A

The head-to-head criterion was not violated. The winner of the election defeated her opponents in a head-to-head comparison.

Ex: The results of an election are summarized in the following preference table. If the plurality method is used to determine the winner, decide whether the head-to-head comparison criterion has been violated and explain your answer.

Number of votes	13	6	1	11
First choice	E	V	E	B
Second choice	V	E	B	V
Third choice	B	B	V	E

The winner by the plurality method is E with 14 votes.

E preferred over V: 1st column (13), 3rd column (1)
$\qquad$ 13 + 1 = 14 voters prefer E over V
V preferred over E: 2nd column (6), 4th column (11)
$\qquad$ 6 + 11 = 17 voters prefer V over E

V wins over E

E preferred over B: 1st column (13), 2nd column (6), 3rd column (1)
$\qquad$ 13 + 6 + 1 = 20 voters prefer E over B

B preferred over E: 4th column (11)
$\qquad$ 11 voters prefer B over E

E wins over B

B preferred over V: 3rd column (1), 4th column (11)
$\qquad$ 1 + 11 = 12 voters prefer B over V
V preferred over B: 1st column (13), 2nd column (6)
$\qquad$ 13 + 6 = 19 voters prefer V over B

V wins over B

Yes. The head-to-head criterion says that if a candidate wins all head-to-head comparisons with all other candidates, then that candidate should win the election. V won all head-to-head comparisons, yet, V did not win the elections. Therefore, the head-to-head criterion has been violated.